Nonmonotonic magnetoresistance of two-dimensional electron systems in the ballistic regime

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We report experimental observations of a novel magnetoresistance (MR) behavior of two-dimensional electron systems in perpendicular magnetic field in the ballistic regime for $k_BT\tau/\hbar > 1$. The MR grows with field and exhibits a maximum at fields $B > 1/\mu$ where μ is the electron mobility. As temperature increases, the magnitude of the maximum grows and its position moves to higher fields. This effect is universal: it is observed in various Si- and GaAs-based two-dimensional electron systems. We compared our data with recent theory based on the Kohn anomaly modification in magnetic field and found qualitative similarities and discrepancies.

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Two-dimensional (2D) degenerate electronic systems of high purity ($k_Fl \gg 1$) with isotropic energy spectrum are rather simple objects, which seem to be well understood. Within the classical kinetic theory,[1] the resistivity of such a system should not depend on perpendicular magnetic field for $\omega_c\tau < 1$ (where $\omega_c = eB/m^*$ is the cyclotron frequency and τ, is the transport time). However, a noticeable magnetoresistance (MR) is often seen in experiments with 2D systems; such MR is usually attributed to quantum corrections which are beyond the classical consideration. There are two types of quantum corrections to conductivity: (i) weak localization (WL) and (ii) electron-electron (e-e) interaction correction (for a review, see Ref. 2). In the diffusive regime ($k_BT\tau/\hbar \ll 1$, $\tau/\tau_\varphi \ll 1$), both corrections give rise to MR with an amplitude proportional to $\ln(T)$.[2,3] The MR should weaken in the crossover regime $k_BT\tau/\hbar \sim 1$ and finally disappear deep in the ballistic regime $k_BT\tau/\hbar \gg 1$.[3]

The theoretical predictions for the MR have been verified in diffusive and diffusive-to-ballistic crossover regimes in recent experiments[4–6] with 2D systems. The ballistic regime, however, was not studied thoroughly. In order to shed light on this issue, we measured MR in the ballistic regime with various simple isotropic 2D electron systems. We have found that the MR in perpendicular fields does not vanish at $k_BT\tau/\hbar > 1$; instead, it manifests a novel type of behavior: *the MR depends nonmonotonically on field and exhibits a maximum, whose position scales with temperature for all samples*.

In this paper, we report observation and systematic studies of the MR in the domain $k_Fl \gg 1$, $k_BT\tau/\hbar > 1$, where the MR should be missing. Experimentally, however, different Si metal-oxide-semiconductor (Si-MOS) structures, GaAs/AlGaAs heterostructures, and GaAs-based quantum wells were found to show a nonmonotonic MR. Our results provide an evidence for a universal origin of the effect. We compared our data with a recent theory[7] of e-e interaction correction that employs modification of the Kohn anomaly by magnetic field and did find some qualitative similarities.

We used two Si-MOS samples (Si4 and Si13 with peak mobilities 1–2 m^2/Vs), two GaAs-AlGaAs heterostructures (28 and 24 with mobilities 21–24 m^2/Vs),[8] gated quantum-well structures AlGaAs-GaAs-AlGaAs (1520, mobility 0.8–1.6 m^2/Vs), and GaAs-InGaAs-GaAs (3513, mobility 2.2 m^2/Vs).[4] All samples were patterned as Hall bars. Density of electrons in gated samples was varied *in situ*. The relevant parameters of the samples are summarized in Table I.

Samples were inserted into a cryostat with a superconducting magnet; the field direction was always perpendicular to the 2D sample plane. Both components of the resistivity tensor were measured simultaneously using the standard four-terminal technique. Current was chosen on order of 1 μA to ensure the absence of electron overheating.

In order to exclude an admixture of the off-diagonal component of the resistivity, we swept magnetic field from $-B$ to B, and then symmetrized the raw data. Such a symmetrization is necessary for reliable measurements of corrections to the resistivity whose relative variations might be less than 1%.

Electron-density values were determined from the slope of the Hall resistance versus B as well as from the period of Shubnikov–de Haas oscillations at low temperatures. Both results agreed with each other within 2%. Temperature was varied in the range of 1.3–60 K. The highest temperature in our experiments was chosen not to exceed 60 K for the carrier density to remain constant and to avoid a bypassing bulk conductivity.

TABLE I. The parameters of the studied samples. Densities are in 10^{12} cm^{-2}, mobilities μ in m^2/Vs, and inverse mean-free times in $k_B\tau/\hbar(1/K)$.

Si sample	n	μ	$\frac{k_B\tau}{\hbar}$	GaAs sample	n	μ	$\frac{k_B\tau}{\hbar}$
Si4	1.3	1.02	0.12	3513	1	2.2	0.11
Si4	1.7	1	0.13	28	0.35	24	1
Si4	2.35	0.96	0.12	24	0.4	21	0.8
Si4	3.4	0.93	0.12	1520	1.6	1.6	0.08
Si13	0.6	2.4	0.29	1520	1.4	1.5	0.07
Si13	0.7	2.3	0.29	1520	1	0.95	0.05
Si13	1	2.1	0.27	1520	0.8	0.8	0.04

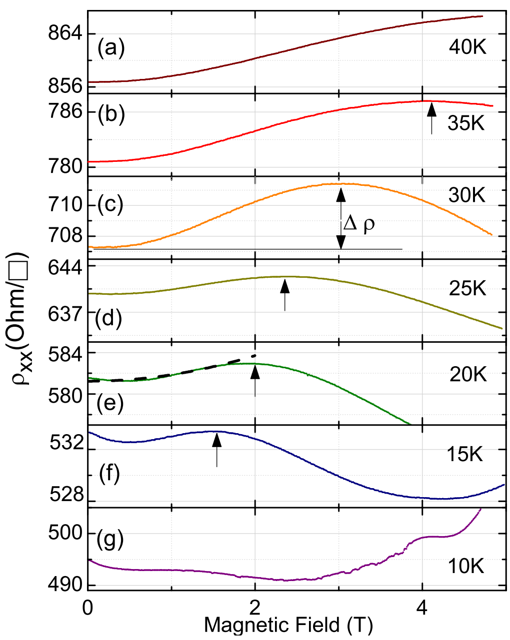

FIG. 1. (Color online) Magnetoresistance for sample Si4 at different temperatures. Electron density $n=1.72\cdot10^{12}$ cm^{-2}. Up arrows mark positions of the ρ_{xx} maxima. $\Delta\rho$ designates the magnitude of the MR. Dashed curve on the panel e shows fitting according to Eq. (2) with $\lambda^2=0.2$. $\hbar/k_B\tau\approx8$ K.

Examples of the MR curves, obtained at different temperatures for samples Si4, 1520, and 28 at fixed electron densities, are shown in Figs. 1–3, respectively. As magnetic field is increased from zero, at first ρ_{xx} sharply falls due to weak localization suppression [see Figs. 1(f), 1(g), and 2]. Then ρ_{xx} starts growing and reaches a maximum at $B^{\max}$ field (indicated by the arrows in Figs. 1 and 3). After passing the maximum ρ_{xx} decreases; in higher fields $|B|>1.5B^{\max}$, MR can become either positive or negative depending on the sample, temperature, electron density, etc. At the lowest temperatures, Shubnikov–de Haas oscillations are seen in high fields, on top of the smooth MR.

The unexpected nonmonotonic magnetoresistance is the main subject of this paper. In different samples and at various electron densities the MR has similar features: (i) it is

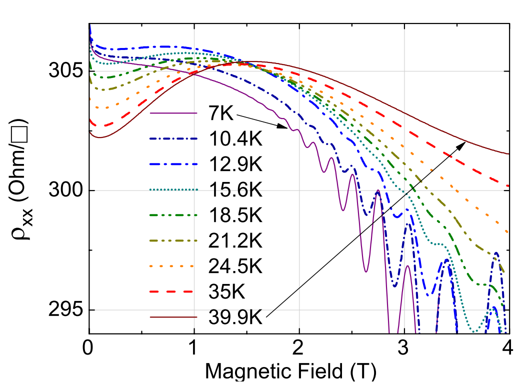

FIG. 2. (Color online) Magnetoresistance for sample 1520 at different temperatures. Electron density $n=1.4\cdot10^{12}$ cm^{-2}. Temperature values are indicated in the figure. $\hbar/k_B\tau=13.5$ K.

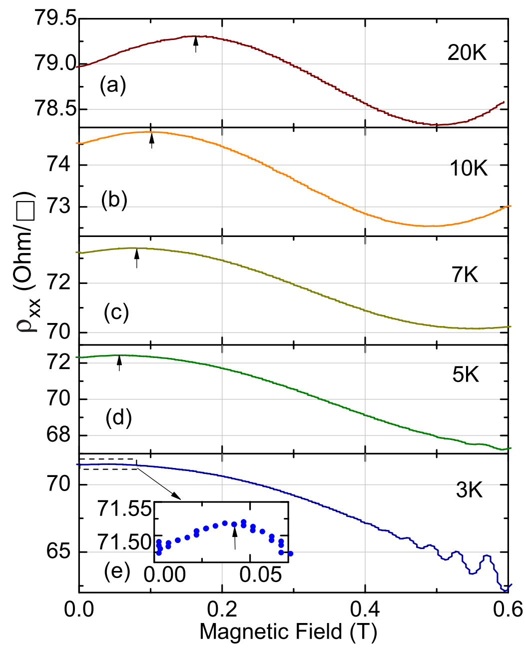

FIG. 3. (Color online) Magnetoresistance for sample 28 at different temperatures. Electron density $n=0.35\cdot10^{12}$ cm^{-2}. Temperature values are indicated in the figure. $\hbar/k_B\tau=1$ K.

small (its typical magnitude is less than 1%), (ii) the nonmonotonic MR is observed only for not-too-low temperatures $T\gtrsim1.3\hbar/k_B\tau$,[9] (iii) the MR maximum grows in magnitude and moves to higher magnetic fields as temperature increases (the position of the maximum exceeds $\omega_c\tau>1$ and is roughly proportional to T).

Comparing the data from Figs. 1 and 2 for Si metal-oxide-semiconductor field-effect transistor (Si-MOSFET) and GaAs-QW samples with similar mobilities and densities, we see that the MR takes a maximum at similar temperatures and magnetic fields, and at similar $\omega_c\tau$ values. This result indicates that the MR has an orbital rather than spin origin because the Zeeman energies $g^*\mu_B$ differ by a factor of 5 for these two different material systems. Also, this effect has nothing to do with WL and e-e-interaction diffusive corrections[2] because it survives at such high temperatures as $k_BT\tau/\hbar\approx20$ for samples 28 and 24 at $T=20$ K.

Searching for possible semiclassical MR mechanisms, we note that most of the theoretical models for short-range scattering (which is the case for all studied samples[10]) predict a *negative, monotonic and temperature independent* magnetoresistance due to the memory effects.[11] A positive T-independent magnetoresistance was predicted in Ref. 12 due to non-Markovian scattering. The latter type of MR was experimentally observed in very clean samples and for classically large magnetic fields[13] $\omega_c\tau\gg1$. Therefore, we conclude that the aforecited mechanisms cannot explain the nonmonotonic MR observed in our experiments.

Recently, Sedrakyan and Raikh[7] suggested a new MR mechanism which causes a maximum of resistivity in not-too-strong magnetic fields $\omega_c\tau\sim1$ in the ballistic regime ($k_BT\tau/\hbar>1$). Therefore, this new mechanism seems to give the best starting point for comparison with our data because all parameter ranges $T\tau>1$, $\omega_c\tau\sim1$, $T/E_F\ll1$, $E_F\tau\gg1$, and the short-range disorder type[10] in experiment are the same as in theory. The MR in Ref. 7 originates from the e-e interaction correction to conductivity, which arises from scattering of electrons on Fridel's oscillations of electron density around impurities.[14,15] Fridel's oscillations are a manifestation of the Kohn $2k_F$ anomaly in screening. In Ref. 16, it was suggested for the first time that even classically weak perpendicular magnetic field ($\omega_c\tau<1$) smears the Kohn anomaly because of the curving of the electron trajectories. It was demonstrated that double scattering from the field-modified Friedel oscillations gives rise to magnetoresistance in the ballistic regime[7]

$$\frac{\delta\sigma_{xx}}{\sigma_{xx}} = 4\lambda^2\left(\frac{\pi k_B T}{E_F}\right)^{3/2} F_2\left(\frac{\omega_c E_F^{1/2}}{2\pi^{3/2}(k_B T)^{3/2}}\right), \tag{1}$$

where $\lambda = 1+3F_0^\sigma/(1+F_0^\sigma)$ is the interaction parameter.[17]

Several predictions can be made based on this equation: (1) the correction to resistivity in small fields is always positive, (2) $(\delta\sigma_{xx}/\sigma_{xx})\cdot(E_F/T)^{3/2}$ is a universal function of $\omega_c E_F^{1/2}/T^{3/2}$ for a given interaction strength λ, and (3) MR has a maximum at $\omega_c\tau \approx 1/\sqrt{3}$.

By comparing these theoretical predictions with our data, we find that prediction (1) is always fulfilled after subtraction of the weak localization. As for prediction (2), the $\rho_{xx}(B)$ data for different temperatures and over the whole range of magnetic fields do not scale as the theory predicts. Furthermore, the position of the MR maximum in our data is temperature dependent and corresponds to $\omega_c\tau \approx 1-3$, contrary to prediction (3). Moreover, in theory, the magnitude of the MR falls as temperature raises whereas *in our experiment it grows with temperature*. Evidently, there is no complete agreement between the theory[7] and our experiment.

In the theory, the maximum of MR inevitably results from $(1-\omega_c^2\tau^2)$ prefactor in resistivity tensor and should occur at $\omega_c\tau \approx 1/\sqrt{3}$. On the other hand, in the experiment the ρ_{xx} maximum is always observed at $\omega_c\tau > 1$, which indicates that this prefactor is weaker than in the theory. Therefore, for the order-of-magnitude comparison, we rewrite Eq. (1) for resistivity by omitting the $[1-(\omega_c\tau)^2]$ prefactor

$$\frac{\delta\rho_{xx}}{\rho_{xx}} = -4\lambda^2\left(\frac{\pi k_B T}{E_F}\right)^{3/2} F_2\left(\frac{\omega_c E_F^{1/2}}{2\pi^{3/2}(k_B T)^{3/2}}\right). \tag{2}$$

Example of fitting the experimental data with a single variable parameter λ^2 is shown in Fig. 1(e). The fit was performed in the limited range of magnetic fields $0.15\omega_c^{\max} < \omega_c < 0.65\omega_c^{\max}$, i.e., in the range of the applicability of Eq. (2) which ignores weak localization and the MR maximum. Contrary to the theoretical expectations, the λ^2 values, i.e., the magnitude of the effect obtained from the fit appeared to be temperature dependent, i.e., grew monotonically from 0.1–0.4 to 1–3 as temperature was increased from $1.3\hbar/(k_B\tau)$ to maximal temperature. Surely this T dependence causes the lack of the scaling predicted by Eq. (1). Moreover, λ^2 values obtained from the fitting do not show a systematic dependence on carrier density and material system. On the other hand, λ^2 may be evaluated from the earlier measurements of $F_0^\sigma(n)$ parameter.[4,18] The calculated λ^2 values are T independent and lie in the interval from 0.2 to 0.5 for GaAs-based structures and from 1.5 to 5 for Si-based structures. We conclude that the observed MR disagrees qualitatively with the theory, although the theory predicts the MR on the right order of magnitude.

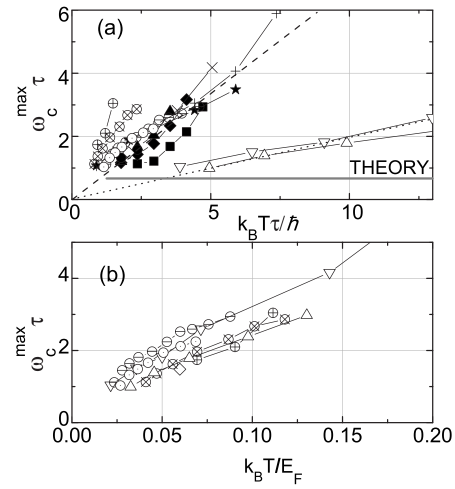

FIG. 4. (a) $\omega_c^{\max}\tau$ value versus dimensionless temperature $k_BT\tau/\hbar$ for all samples. Electron densities (in units of 10^{12} cm^{-2}) are ★-n=0.6 (Si13); +-n=0.7 (Si13); ×-n=1 (Si13); ■-n=1.3 (Si4); ◆-n=1.7 (Si4); ▲-n=2.35 (Si4);•-n=3.4 (Si4); ⊙-n=1.4 (1520); ⊗-n=1 (1520); ⊖-n=1.6 (1520); ⊕-n=0.8 (1520); ◇-n=1 (1520); ▽-n=0.35 (28); and △-n=0.4 (24). Dashed line corresponds to $\hbar\omega_c^{\max}=0.7k_BT$. Dotted line is $\hbar\omega_c^{\max}=0.2k_BT$. Horizontal thick line is the theoretical prediction (see in the text). (b) The same data for GaAs-based samples solely scale in coordinates $\omega_c^{\max}\tau$ versus k_BT/E_F.

Evidently, there is a lack of consistency between the experiment and the theory in terms of the theory parameters. Therefore, using a phenomenological approach we search for such characteristics of the MR which scale with relevant parameters of the problem. First, we analyze the magnitude of the effect. As a rough estimate of the magnitude, we introduce $\Delta\rho$, a difference between $\rho_{xx}(B^{\max})$ and minimal ρ_{xx} at $B<B^{\max}$ [see in Figs. 1(d) and 1(f)]. Despite a certain arbitrariness, thus defined magnitude is almost unaffected by the WL and always grows as temperature increases. Unfortunately, we found that $\Delta\rho$ does not scale with dimensionless combinations of any of the energy-related parameters k_BT, $\hbar/\tau$, $\hbar\omega_c^{\max}$, and E_F and hence, is not an appropriate characteristic of the effect.

We now turn to the position of the MR maximum $\omega_c^{\max}$ which is plotted in Fig. 4(a) as a function of temperature. The $\omega_c^{\max}\tau$ value systematically exceeds the theoretical expectation $1/\sqrt{3}$ [horizontal thick line in Fig. 4(a)] and approximately equals $0.7k_BT\tau/\hbar$ for most of the data [dashed curve in Fig. 4(a)]. For samples with the highest mobility (24 and 28), the slope $\omega_c^{\max}\tau/(k_BT\tau/\hbar)$ approximately equals 0.2, whereas for GaAs-based sample with the lowest mobility the slope exceeds 0.7. In order to take this fact into account, we have applied another scaling in coordinates $\omega_c^{\max}\tau$ versus k_BT/E_F [see Fig. 4(b)]. Remarkably, for GaAs-based 2D systems with mobilities and conductivities ranging by more than an order of magnitude, the $\omega_c^{\max}\tau$ data indeed scale reasonably, the result that might suggest a clue for understanding the effect.

The data for Si-based structures are not shown in Fig. 4(b) because they fall out of the T/E_F scaling. In order to understand the origin of the difference in scaling for Si and GaAs samples, we note that for GaAs-based samples in high fields $B > B^{max}$, the MR is always negative while for Si-based samples it can be either negative or positive, depending on particular sample and electron density. It means that some other mechanisms affect MR in Si-MOSFETs in strong perpendicular fields $B > B^{max}$ and shift the MR maximum. Of particular importance may be the temperature dependence of scattering time $\tau(T)$ which is strong in Si-MOSFETs,[18] as seen from Fig. 1. In theory,[7] τ was assumed to be temperature independent, which may partially account for the discrepancy between Si- and GaAs-based samples.

We note also that due to clear reasons, the nonmonotonic MR in the ballistic regime $T \geq 1.3\hbar/k_B\tau$ was not observed in the following cases: (i) Si-MOSFETs in the domain of strong interactions ($n < 6 \cdot 10^{11}$ cm^{-2}) where the giant negative MR develops and dominates over other weak effects,[19] (ii) Si-MOSFETs for such high temperatures where Fermi gas is nondegenerate ($T/E_F \geq 0.5$), (iii) GaAs-based samples at such high temperatures that the carrier density becomes B and T dependent.

In this paper we report experimental observation of the novel nonmonotonic behavior of the magnetoresistance for 2D electron systems in perpendicular field. This MR is intrinsic to various 2D systems (Si-MOSFETs, GaAs, and InGaAs quantum wells, and GaAs/AlGaAs heterostructures) and occurs in the ballistic regime of high temperatures $T\tau > 1$. The MR is positive in low fields and reaches a maximum at $\omega_c\tau = 1-3$; the position of the maximum ω_c^{max} scales linearly with temperature for all samples.

We compare our data with recently suggested MR mechanism[7] and find some similarities: (i) the MR is always positive in low field, (ii) the MR exhibits a maximum in higher field, and (iii) the MR is on the same order of magnitude as predicted. We believe, therefore, that the theory[7,16] may serve as a basis for further development of the MR theory. However, some other features of our experimental data are in discrepancy with the theory of Ref. 7: (i) the MR maximum is achieved in fields which are noticeably higher than predicted, (ii) the position of the MR maximum linearly depends on temperature rather than remains constant, and (iii) the magnitude of the effect increases with temperature rather than decreases, as predicted.

Some clue to understanding the effect may be provided by scaling of the MR maximum position $\omega_c^{max}\tau$ versus T/E_F, which is empirically observed for various GaAs samples in wide ranges of temperature, density, and mobility. The observation of the nonmonotonic MR shows that the magnetotransport theory for the ballistic regime requires further consideration.

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[9] Throughout this paper the following notations are used: $E_F = p_F^2/2m^*$, $\omega_c = eB/m^*$, and $\tau = \sigma_D m^*/ne^2$, where m^* is the electron-band mass ($0.067m_e$ for GaAs and $0.21m_e$ for Si) and σ_D is the Drude conductivity. The latter is found by extrapolating the conductivity in ballistic regime to $T=0$.

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